Department of Physics
St. Petersburg State University

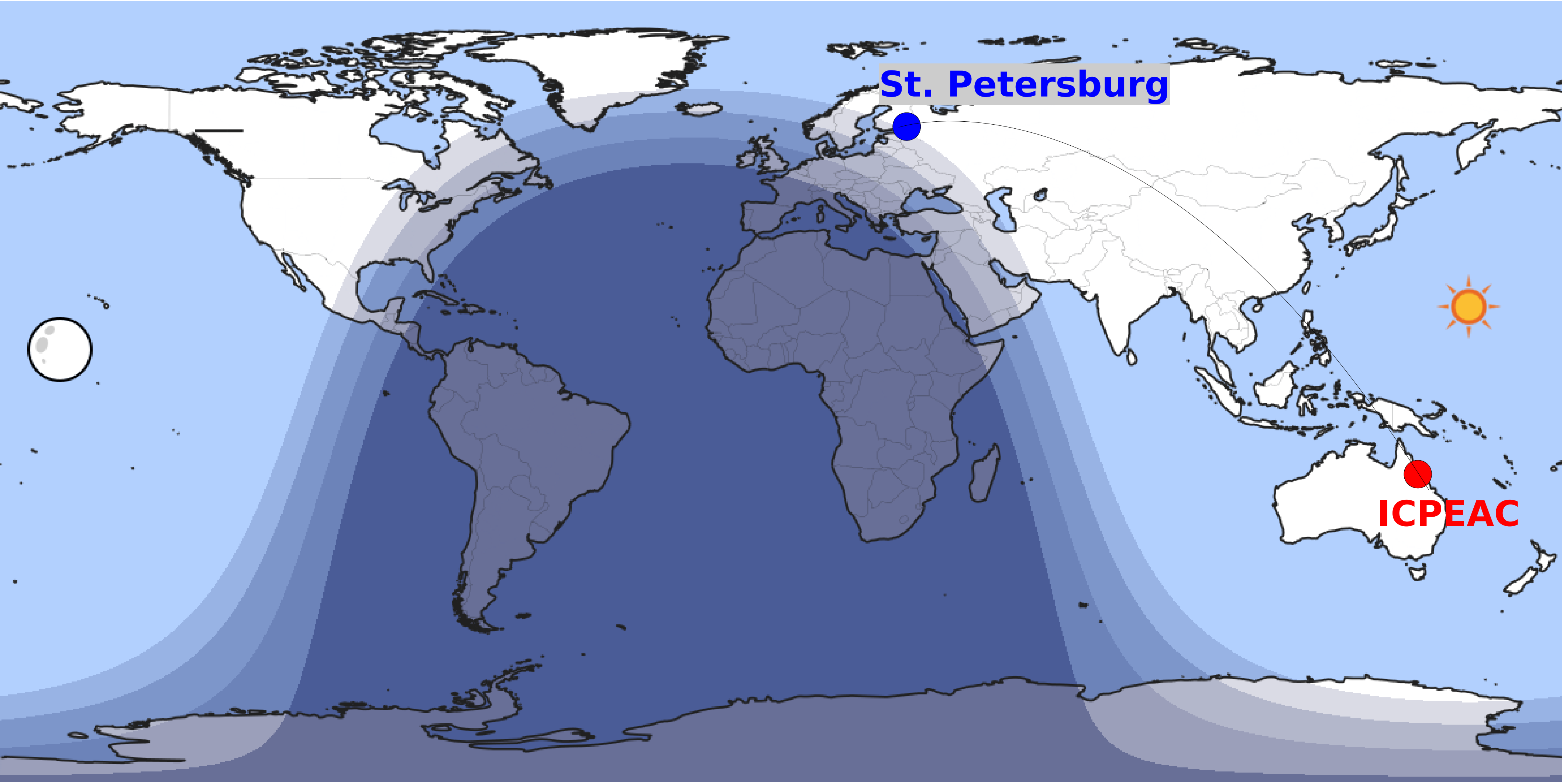
Pair creation in low-energy heavy ion-atom collisions

Ilya Maltsev, Yury Kozhedub, Roman Popov, Adnrey Bondarev,
Ilya Tupitsyn, Vladimir Shabaev,
and Thomas Stöhlker

Outline

- Intoduction and Motivation
- Theoretical Description and Numerical Results
- Summary

Day and Night World Map

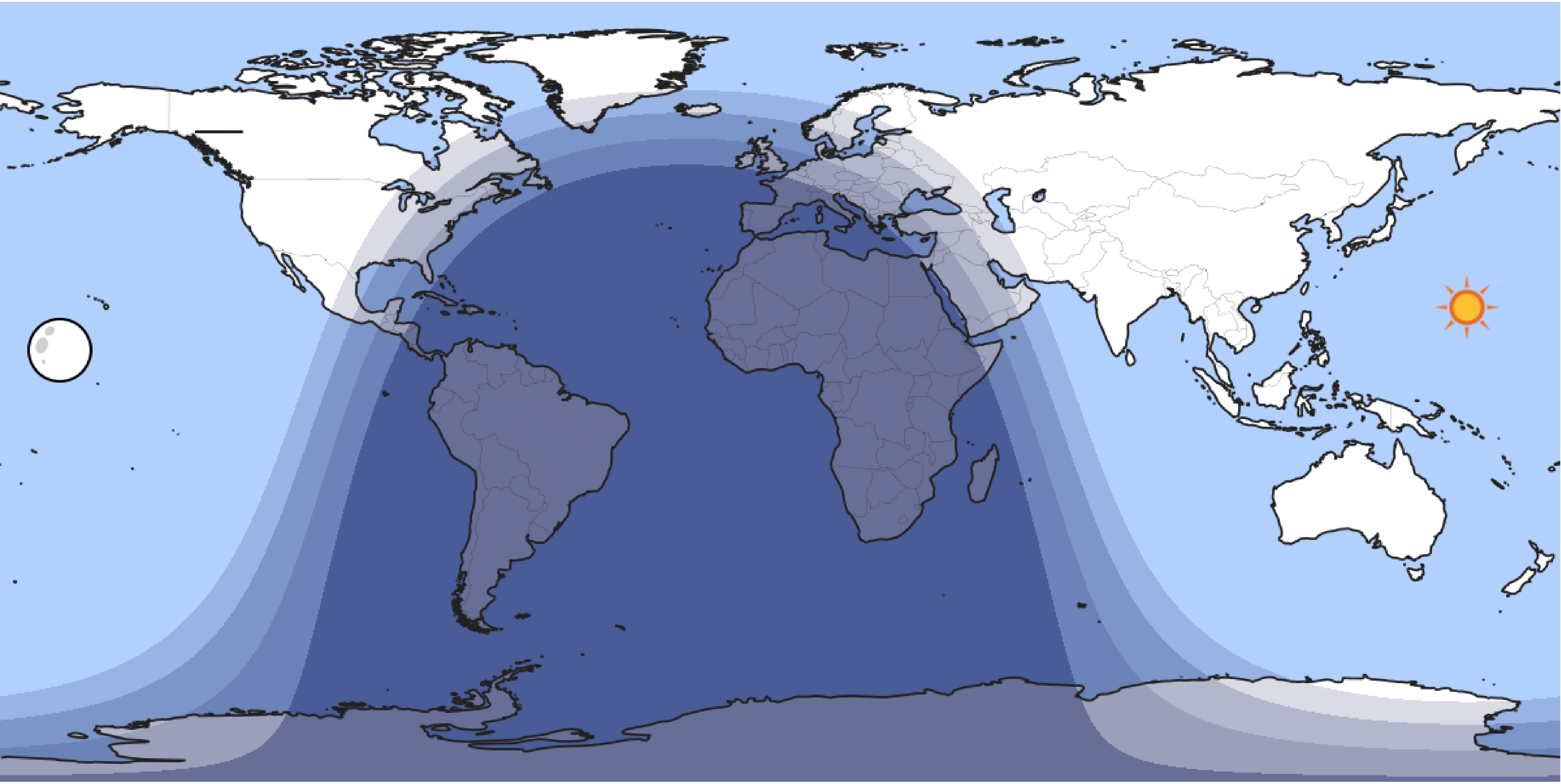


Distance: ~13 000 km

Cairns, Australia: 11:32 a.m.
St. Petersburg, Russia: 4:32 a.m.

Coastline

The length of the coast	Russia:	38 800 km
	Australia:	35 900 km



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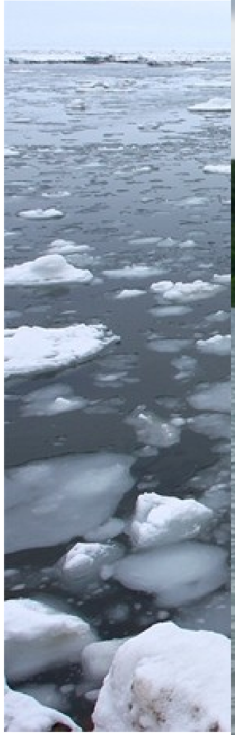

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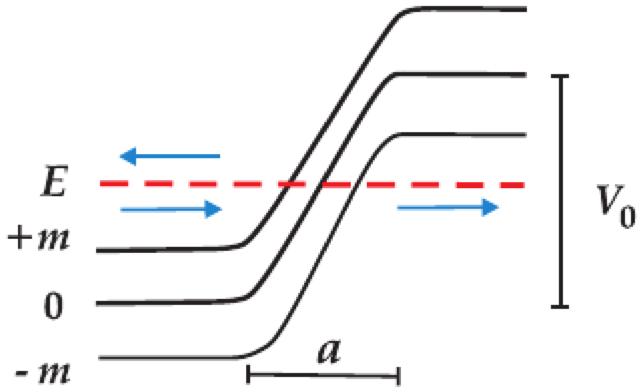
Strong Fields in QED - History

- **Klein's paradox and pair creation**



Klein (1929): Anomalous transmission and reflection coefficients at a potential barrier with $V_0 > 2m$.

Euler + **Heisenberg** (1936), **Weisskopf**: Nonlinear effective QED Lagrangian for homogeneous *E* and *B* fields. Imaginary part −→ pair production.

Schwinger formula (1951): $2\,\mathrm{Im}\,L_{\text{eff}} = \frac{eE}{(2\pi)^3}\sum_{n=1}^{\infty}\frac{1}{n}e^{-nE_{cr}/E}$

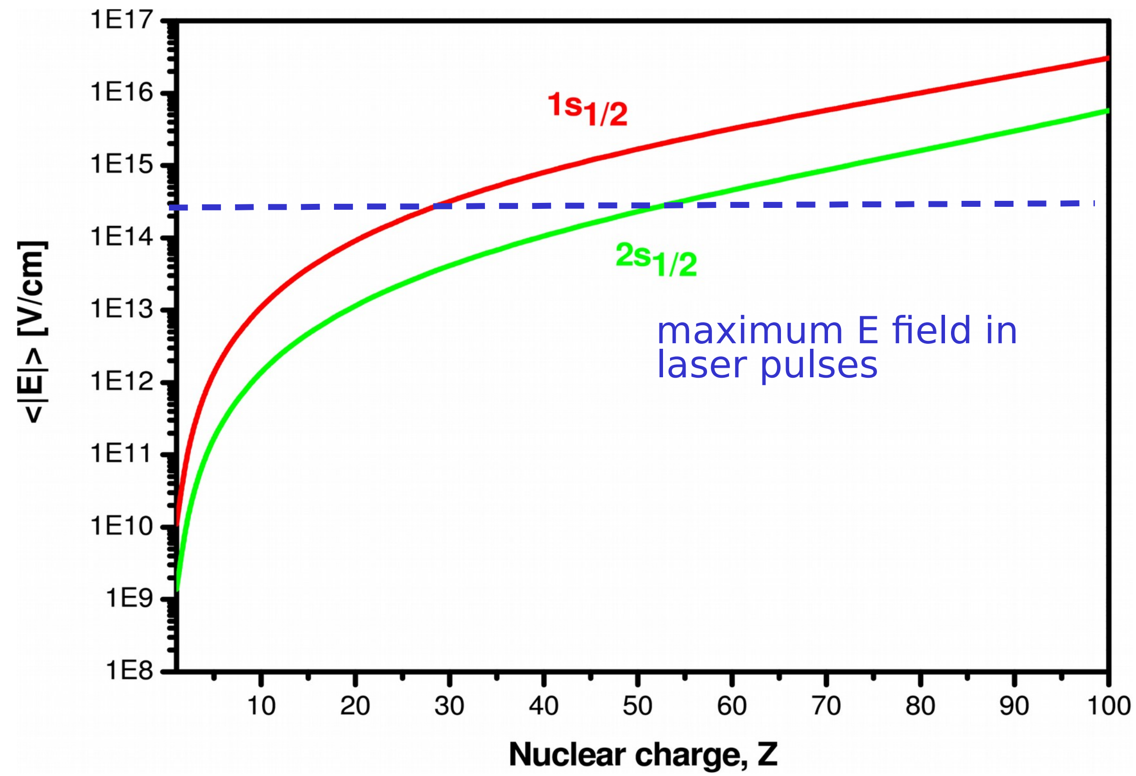
Critical field strength: $E_{cr} = \frac{\pi m^2 c^3}{e\hbar} \sim 10^{16}\frac{V}{cm}$

Nikishov (1970), **Gavrilov** + **Gitman** (2016): Rigorous QFT explanation

Strong Fields in QED - History

- **Supercritical atoms**

Heavy few-electron ions
are sources of extremely strong
electromagnetic fields.



Sommerfeld formula:

$E_{1s} = mc^2 \sqrt{1-(\alpha Z)^2}$ "Collapse of the wavefunction" at $Z = 1/\alpha \simeq 137.$

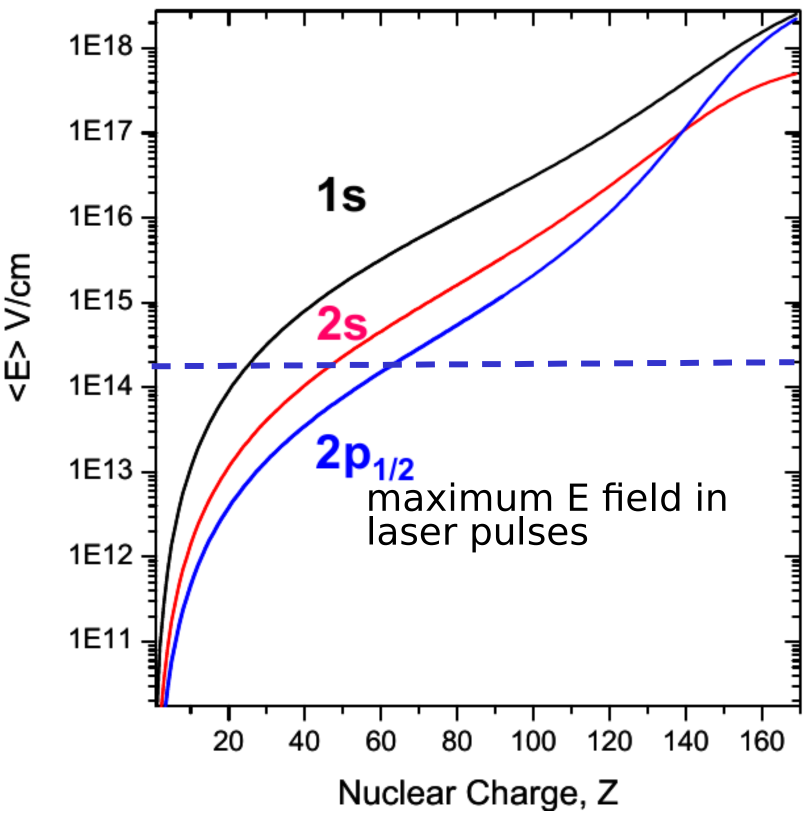
Finite nuclei:

nuclear charge distribution: Pomeranchuk, Smorodinskij (1945)

finite nuclear mass: Aleksandrov et al. (2016)

Strong electromagnetic fields

- **Supercritical atoms**



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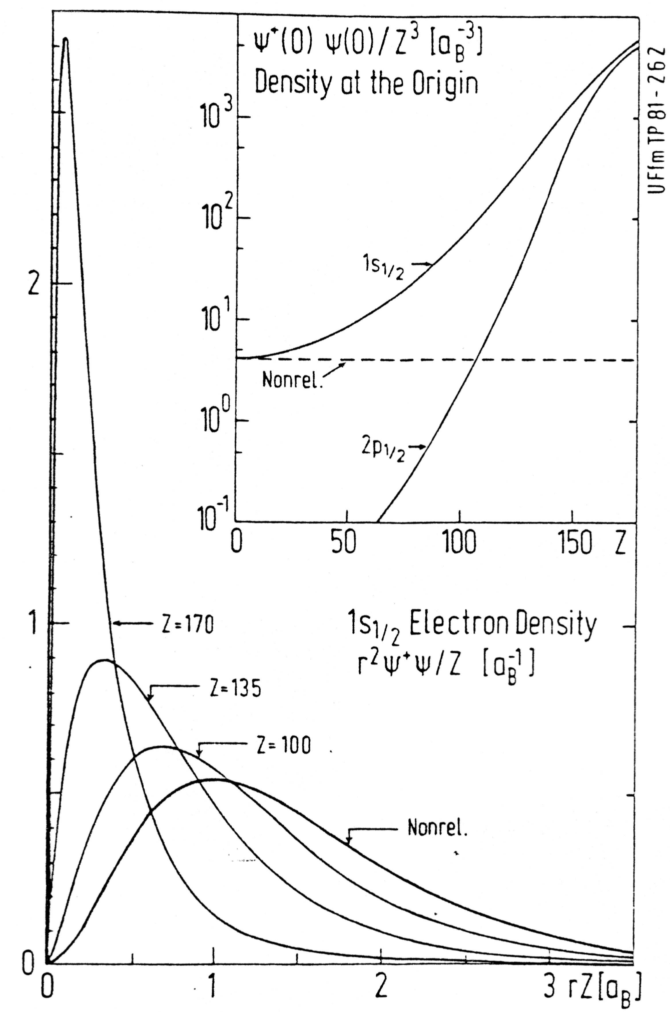
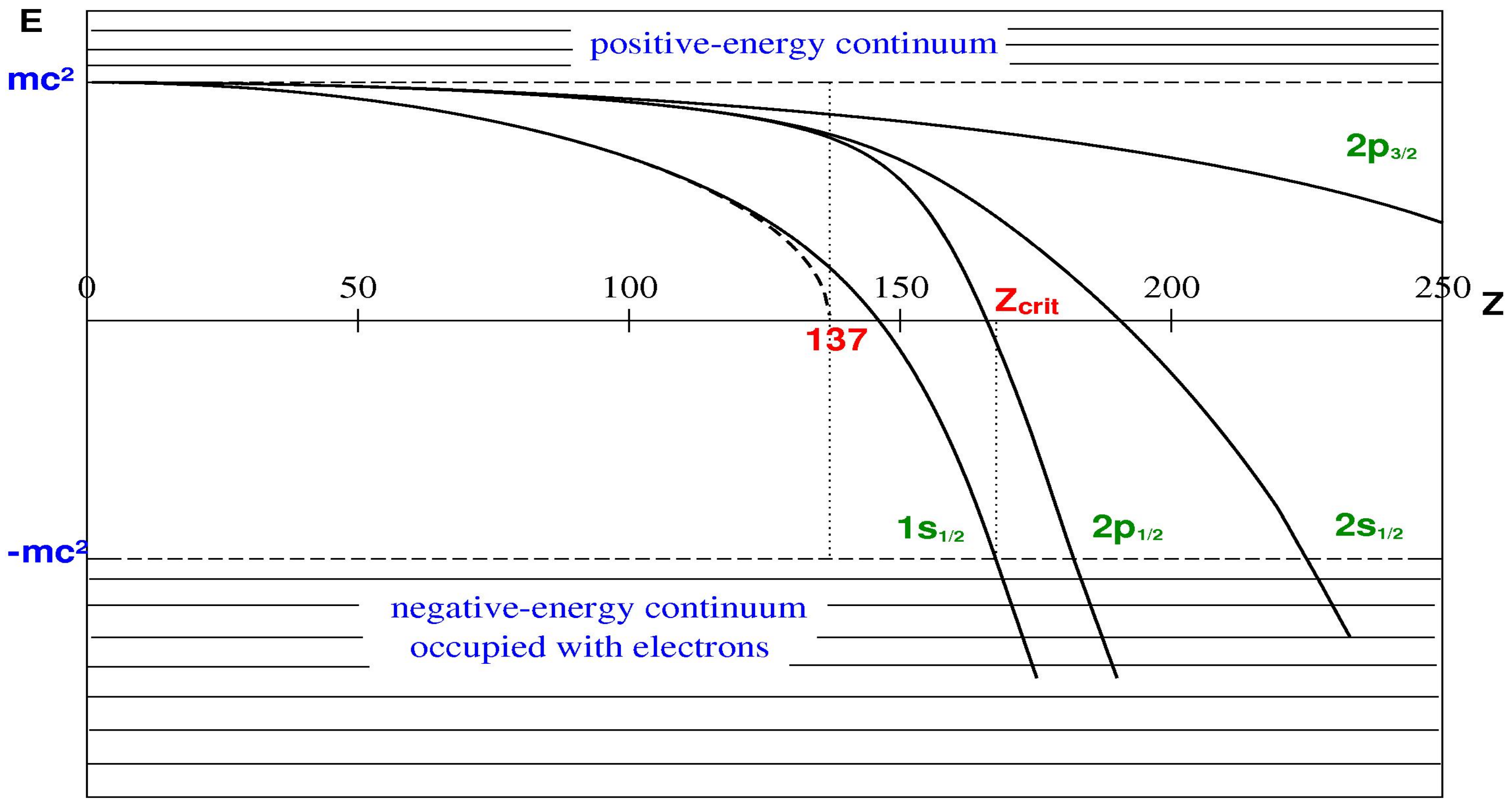


Tremendous increasing of the radial density distribution for the $2p_{1/2}$ wave function by more than four orders of magnitude when Z increases from 60 to 173.

Strong electromagnetic fields

- **Supercritical atoms**

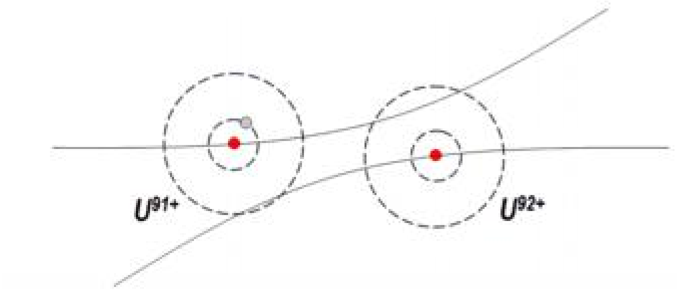
Gershtein, Zeldovich (1969); Pieper, Greiner (1969)

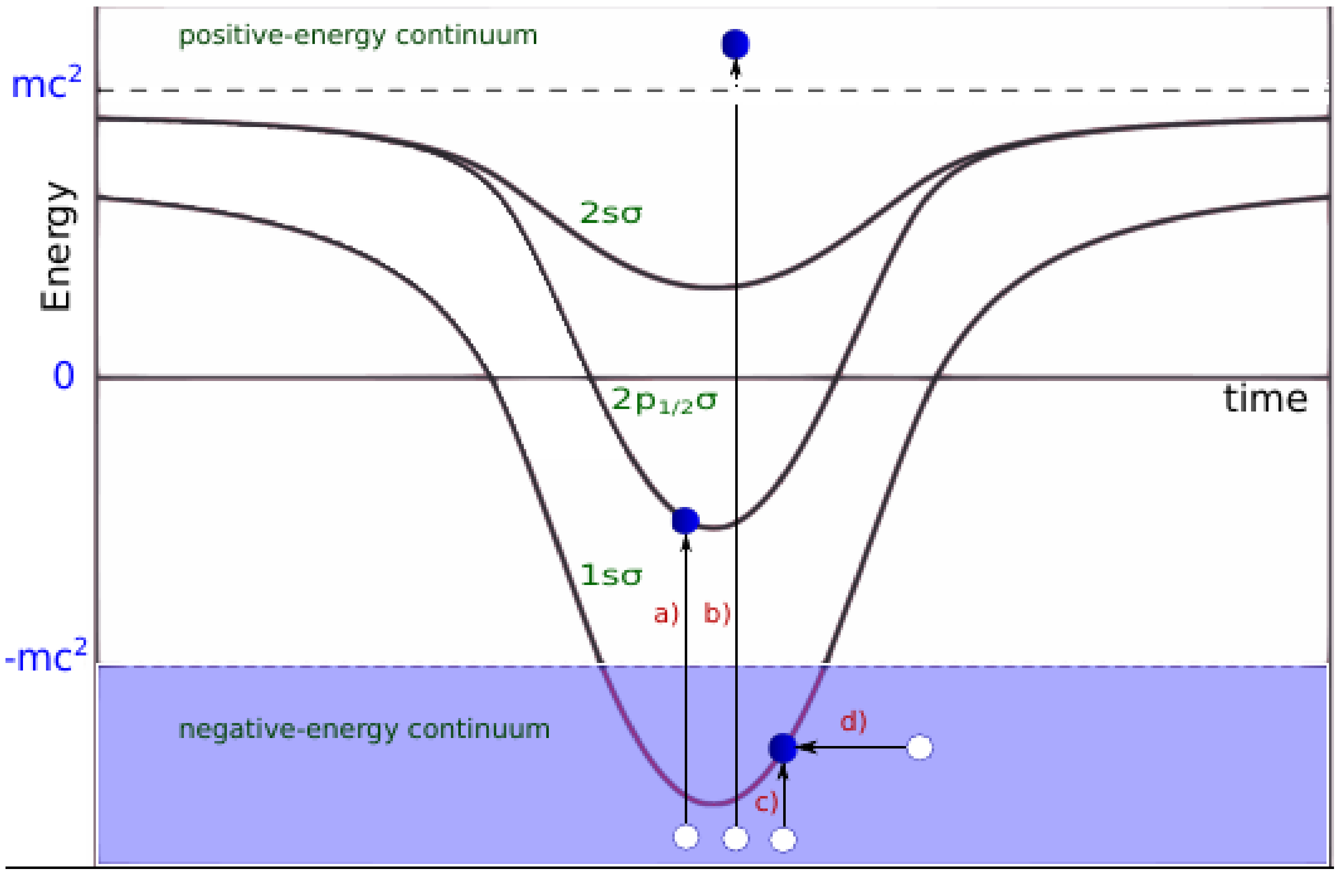


The 1s level dives into the negative-energy continuum at Z_{crit} ~173.
Spontaneous pair creation. Charged vacuum.

Strong electromagnetic fields in heavy ion collisions

Supercritical field could be achieved in collisions of two heavy ions with $Z_1+Z_2 > 173$.





1970-80s

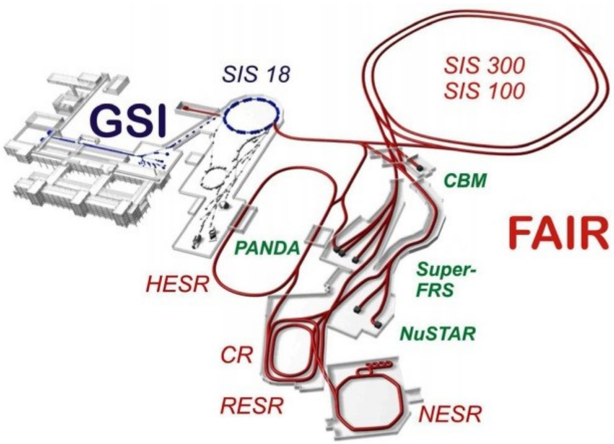
Frankfurt group:
Greiner, Müller, Soff,
Reinhardt, Müller-Nehler, ...

Moscow:
Popov

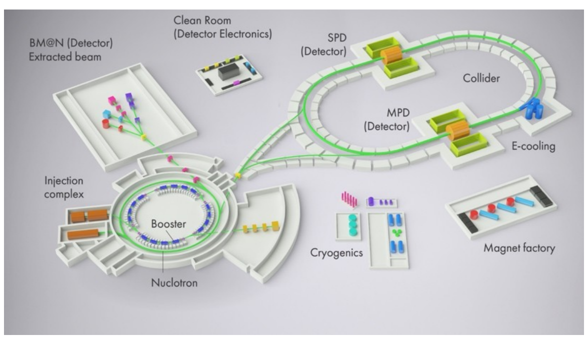
GSI (Darmstadt):
collisions of partially stripped ions with neutral atoms

Dynamic (induced) pair creation a), b), c)
Spontaneous pair creation d)

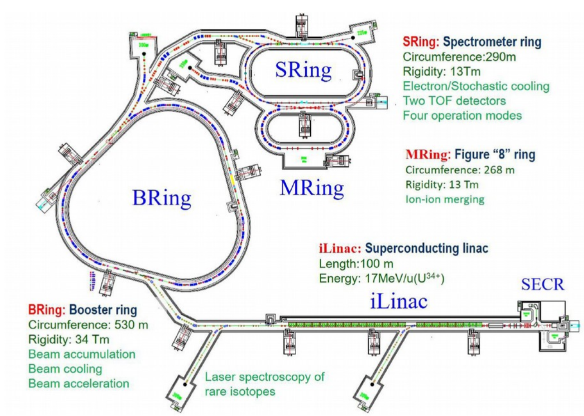
Heavy ion collisions: future experimental facilities



Facility for Antiproton and Ion Research (**FAIR**),
Germany

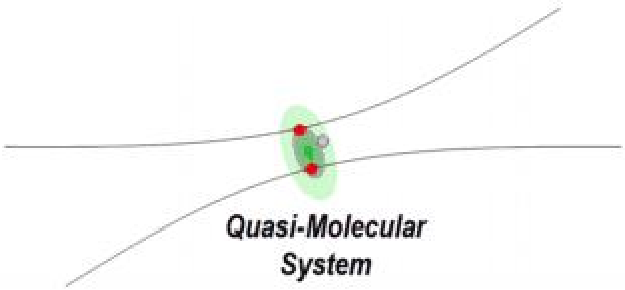


Nuclotron-based Ion Collider fAcility (**NICA**),
Russia



High Intensity heavy ion Accelerator Facility (**HIAF**),
China

Heavy ion collisions



- The most favorable energy is about Coloumb barrier (Low-energy collision ~ 6 MeV/u for U).

- Semiclassical approximation: R(t) = Rutherford trajectory.

- Two-Center Dirac Hamiltonian:

$$H_{TCD}(\vec{R}) = c\,\vec{\alpha}\,\vec{p} + m_e c^2 \beta + V_1(\vec{r}, \vec{R}(t)) + V_2(\vec{r}, \vec{R}(t)).$$

- Nonperturbation approach.

- Diving time period is about 10^{-21} sec.
  Spontaneous e^+e^- pair creation time is about 10^{-19} sec
  [Müller et al., 1972, Ackad and Horbatsch, 2008].

- $R_{crit}(1\sigma_+) = 34.7$ fm (for U + U)
  $R_{nucl} \quad = \quad 5.9$ fm

Previous theoretical study

Quasistationary approach

Only *spontaneous* pair creation is taken into account.

- H. Peitz, B. Müller, J. Raeski, and W. Greiner, Lett. Nuovo Cimento 8, 37 (1973)
- V. S. Popov, Sov. Phys. JETP 38, 18 (1974)

Perturbative approach

Only *dynamical* pair creation is taken into account.

- R. N. Lee and A. I. Milstein, Phys. Lett. B 761, 340 (2016)
- I. B. Khriplovich, Eur. Phys. J. Plus 132, 61 (2017)

Dynamical approach

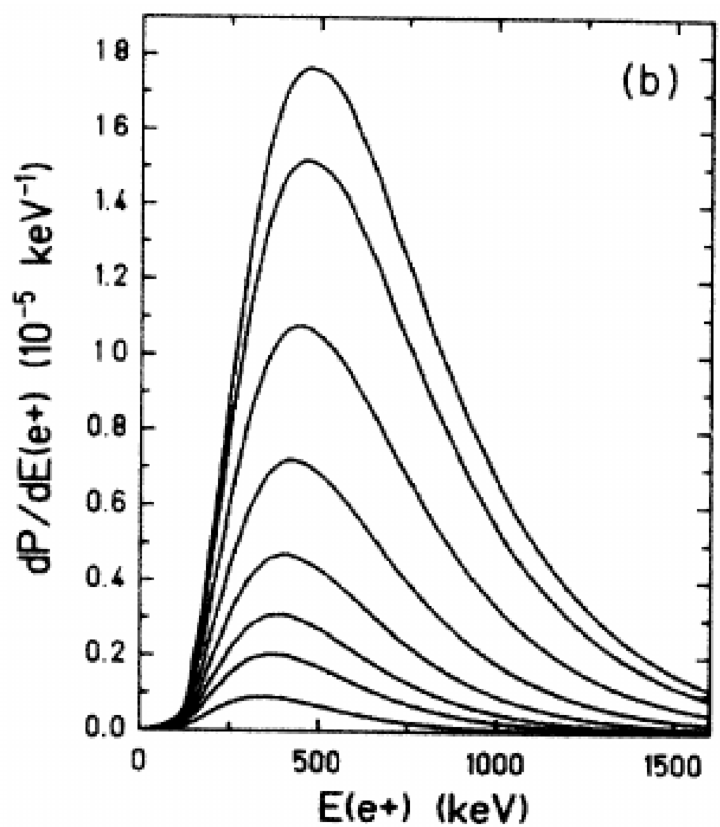
Both *dynamical* and *spontaneous* contributions are taken into account.
The approach is based on the numerical solving of the time-dependent Dirac equation in the monopole approximation.

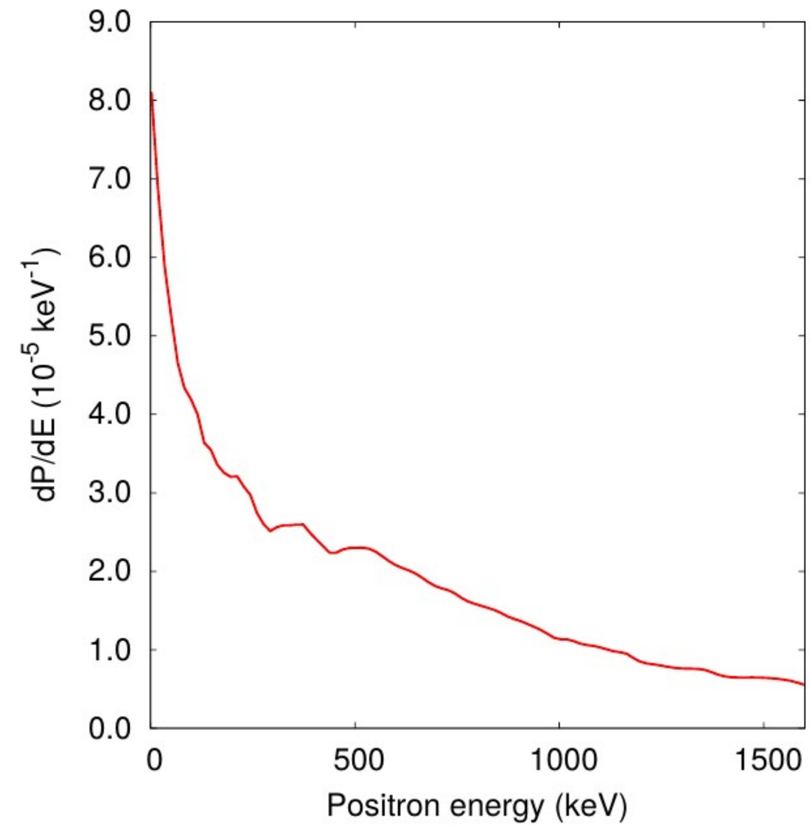
- Frankfurt group
  U. Müller et al., PRA 37, 1449 (1988)

- Canadian group
  E. Ackad and M. Horbatsch, PRA 78, 062711 (2008)

Previous theoretical study: disagreement

Positron energy spectrum for U^{92+}-U^{92+} collisions at E=6.2 MeV/u for the impact parameter b





b =0, 5, 10, 15, 20, 25, 30, 40 fm
U. Müller et al., PRA 37, 1449 (1988)

b =0 fm
E. Ackad and M. Horbatsch, PRA 78, 062711, (2008)

In/out formalism

$$i\frac{d\,\psi(\vec{r},t)}{dt}=H_{\mathrm{TCD}}(t)\,\psi(t),$$

$$H_{\mathrm{TCD}}(t)=c\,\vec{\alpha}(\vec{p}\,\frac{e}{c}\vec{A}(t))+V(t)+m_e c^2\beta.$$

Two sets of solutions: $\psi_i^{+}(\vec{r},t_{\mathrm{in}})=\psi_i^{\mathrm{in}}(\vec{r}),\quad \psi_i^{-}(\vec{r},t_{\mathrm{out}})=\psi_i^{\mathrm{out}}(\vec{r}),$

- $t_{\mathrm{in}}\rightarrow-\infty$ is the initial time moment,
- $t_{\mathrm{out}}\rightarrow\infty$ is the final time moment,
- $H_{\mathrm{TCD}}(t_{\mathrm{in}})\,\psi_i^{\mathrm{in}}(\vec{r})=\epsilon_i^{\mathrm{in}}\,\psi_i^{\mathrm{in}}(\vec{r}),\quad H_{\mathrm{TCD}}(t_{\mathrm{out}})\,\psi_i^{\mathrm{out}}(\vec{r})=\epsilon_i^{\mathrm{out}}\,\psi_i^{\mathrm{out}}(\vec{r}).$

Pair production in external field

Number of electrons in k state: $$n_k = \sum_{j<F} |a_{kj}|^2 .$$

Number of positrons in p state: $$\bar{n}_k = \sum_{j>F} |a_{pj}|^2 .$$

Here F is the Fermi level ($\varepsilon_F = -m_e c^2$) and

$$a_{ij} = \int d^3\vec{r}\, \psi_i^{(-)\dagger}(\vec{r},t)\, \psi_j^{(+)}(\vec{r},t) = \int d^3\vec{r}\, \psi_i^{(\mathrm{out})\dagger}(\vec{r})\, \psi_j^{(+)*}(\vec{r},t_{\mathrm{out}}).$$

Forward time propagation:

$$a_{ij} = \int d^3\vec{r}\, \psi_i^{(\mathrm{out})\dagger}(\vec{r})\, \psi_j^{(+)*}(\vec{r},t_{\mathrm{out}}).$$

Backward time propagation:

$$a_{ij} = \int d^3\vec{r}\, \psi_i^{(-)\dagger}(\vec{r},t_{\mathrm{in}})\, \psi_j^{(\mathrm{in})}(\vec{r}).$$

Monopole approximation

- Time-dependent two-center potential of moving nuclei:

$$V(\vec{r},t)=V_{\text{nucl}}^{(A)}(\vec{r}-\vec{R}_A(t))+V_{\text{nucl}}^{(B)}(\vec{r}-\vec{R}_B(t)).$$

- In the monopole approximation only spherically symmetric part of $V(\vec{r},t)$ is taken into account:

$$V_{\text{mon}}(r,t)=-\frac{1}{4\pi}\int d\Omega\, V(\vec{r},t).$$

It allows reducing the three-dimensional two-center Dirac equation to one-dimensional radial one.

Finite basis expansion

$$\phi(r,t)=\sum_i C_i(t)u_i(r)$$

$\phi(r,t)$ is the radial part of the wave function

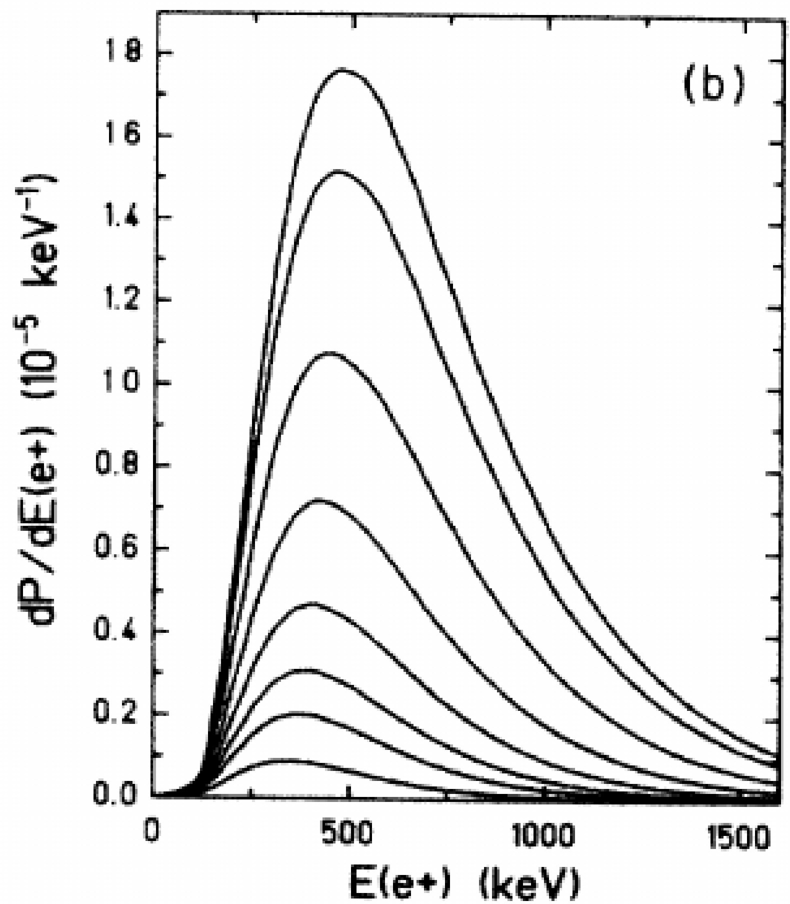
$u_i(r)$ are the basis functions constructed from B-splines using DKB technique (Shabaev et al., PRL (2004)) which prevents the appearance of spurious states

$$i\sum_{i=1}^{N} S_{ji}\frac{dC_i(t)}{dt}=\sum_{i=1}^{N} H_{ji}C_i(t)$$

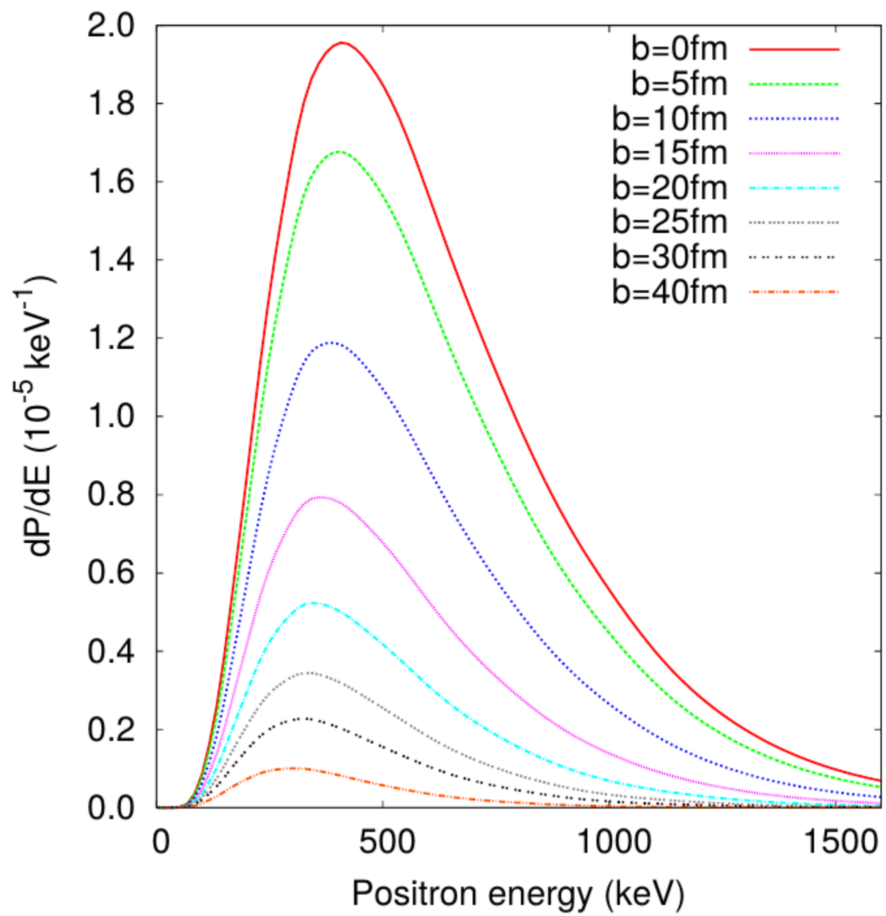
$S_{ij}=\langle u_i|u_j\rangle$, $H_{ij}=\langle u_i|H(t)|u_j\rangle$, $H(t)$ is the radial Hamiltonian

Positron energy spectrum

U^{92+}-U^{92+} collisions at E=6.2 MeV/u for the impact parameters b=0-40fm



U. Müller et al., PRA 37, 1449 (1988)



I.A. Maltsev et al., PRA 91, 032708 (2015)

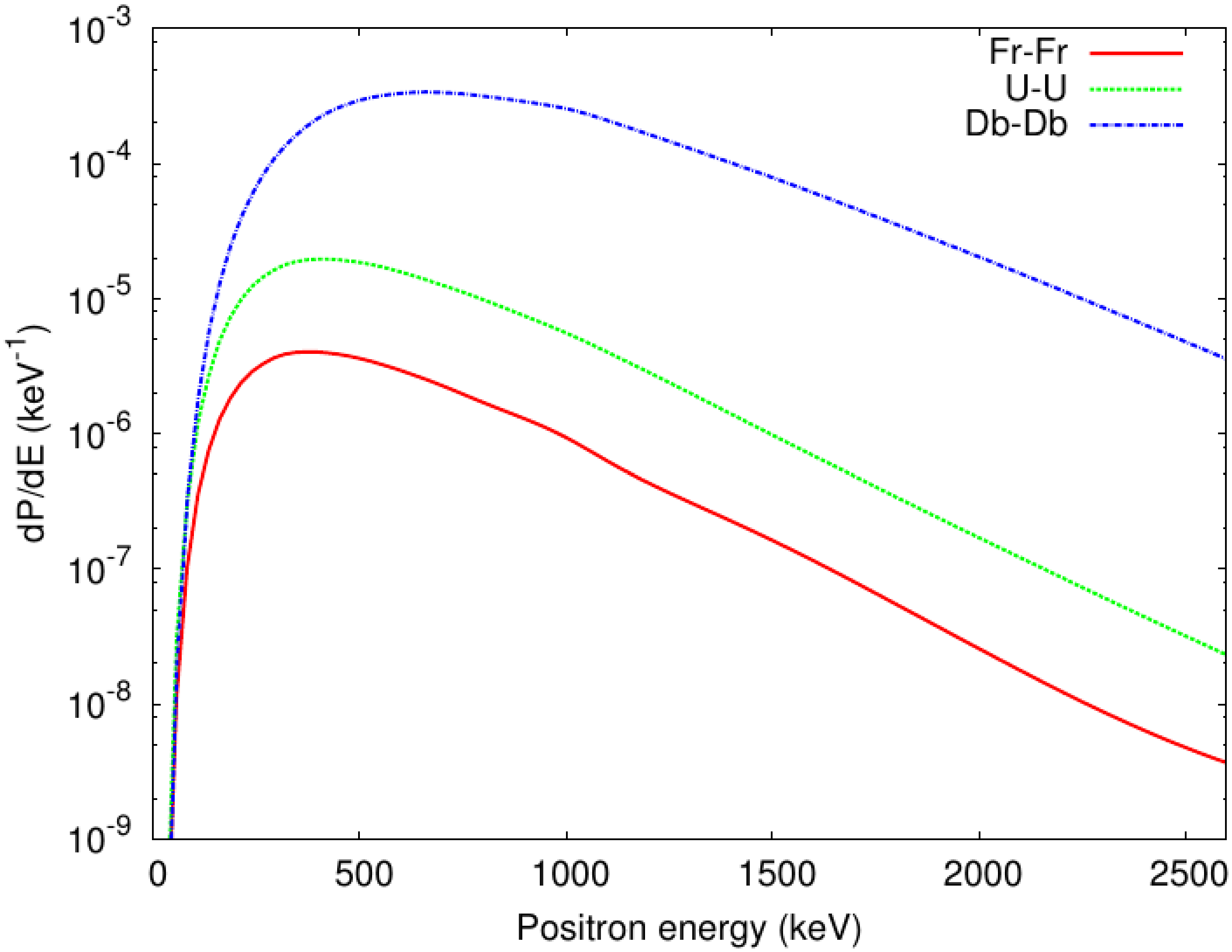
Pair-production probabilities for E_{cm} =740 MeV

P_t is the total probability

P_b is the probability of pair production with an electron in a bound state

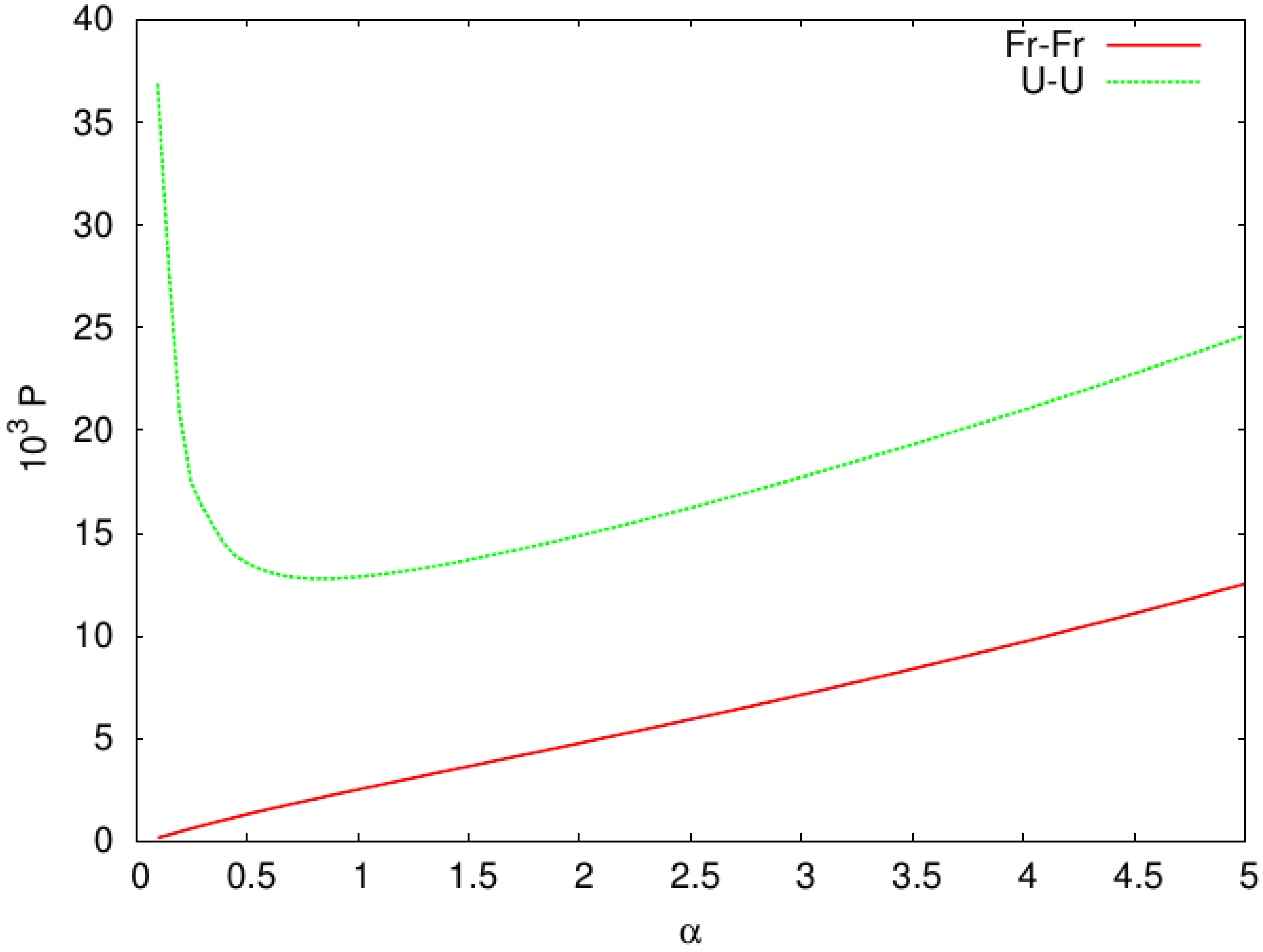
	Müller et al.		This work	
b (fm)	P_b	P_t	P_b	P_t
0	1.23×10^{-2}	1.26×10^{-2}	1.25×10^{-2}	1.29×10^{-2}
5	1.04×10^{-2}	1.06×10^{-2}	1.05×10^{-2}	1.08×10^{-2}
10	7.04×10^{-3}	7.15×10^{-3}	7.03×10^{-3}	7.26×10^{-3}
15	4.41×10^{-3}	4.47×10^{-3}	4.39×10^{-3}	4.51×10^{-3}
20	2.71×10^{-3}	2.73×10^{-3}	2.70×10^{-3}	2.75×10^{-3}
25	1.67×10^{-3}	1.68×10^{-3}	1.66×10^{-3}	1.69×10^{-3}
30	1.04×10^{-3}	1.04×10^{-3}	1.03×10^{-3}	1.04×10^{-3}
40	4.11×10^{-4}	4.11×10^{-4}	4.09×10^{-4}	4.12×10^{-4}

Positron energy spectrum



Positron energy spectrum for the Fr− Fr (Z=87), U−U (Z=92), and Db−Db (Z=105) head-on collisions at energies 674.5, 740, and 928.4 MeV, respectively (I.A. Maltsev et al., PRA 91, 032708 (2015)).

Collisions with modied velocity



Number of created pairs P in the head-on collision with modied velocity:

$$\dot{R}_\alpha(t) = \alpha \dot{R}(t).$$

I.A. Maltsev et al., PRA 91, 032708 (2015)

Beyond the monopole approximation: two-center Dirac equation in rotating coordinates

$$i\frac{d\psi}{dt}=H_{\mathrm{D}}\psi-\vec{J}\vec{\omega}\psi$$

$$H_{\mathrm{D}}=c\vec{\alpha}\vec{p}+V+\beta m_e c^2$$

$$V=V_{\mathrm{nucl}}^{(A)}(\vec{r}-\vec{R}_A(t))+V_{\mathrm{nucl}}^{(B)}(\vec{r}-\vec{R}_B(t))$$

$\vec{\omega}$ is the angular velocity of the reference frame

$\vec{J}$ is the operator of the total angular momentum

$\vec{J}\vec{\omega}$ breaks the axial symmetry

Beyond the monopole approximation: basis expansion

$$\psi(r,\theta)=\sum_n C_n(t)W_n(r,\theta)$$

The rotational term $\vec{J}\vec{\omega}$ is neglected

$\psi(r,\theta,t)$ is the four-component wave function

$W_n(r,\theta)$ are constructed from the B-splines using the DKB technique for axially symmetric systems [Rozenbaum et al., PRA 89, 012514 (2014)]

Beyond the monopole approximation: results

U-U, Ecm = 740 MeV

b is the impact parameter.
P_b is the probability of pair creation with an electron in any bound state.
P_g is the probability of pair creation with an electron in the ground state of the quasi-molecule.

	Two centers		Monopole
b (fm)	P_b	P_g	P_b
0	1.32×10^{-2}	1.11×10^{-2}	1.25×10^{-2}
5	1.11×10^{-2}	9.44×10^{-3}	1.05×10^{-2}
10	7.62×10^{-3}	6.56×10^{-3}	7.03×10^{-3}
15	4.87×10^{-3}	4.27×10^{-3}	4.39×10^{-3}
20	3.07×10^{-3}	2.73×10^{-3}	2.70×10^{-3}
25	1.92×10^{-3}	1.75×10^{-3}	1.66×10^{-3}
30	1.24×10^{-3}	1.13×10^{-3}	1.03×10^{-3}
40	5.33×10^{-4}	4.84×10^{-4}	4.09×10^{-4}

Summary

- The pair-production process in low-energy collisions of bare nuclei has been investigated in the monopole approximation (I.A. Maltsev et al., PRA 91, 032708 (2015)).
- The obtained results for U −U collisions are in good agreement with the corresponding values from Müller et al., PRA 37, 1449 (1988).
- The method for calculation of pair-creation process beyond the monopole approximation has been developed.
- The first results for the full two-center potential have been obtained.

Poster listing

- **FR-110 Electron-positron pair creation in collisions of heavy bare nuclei: One-center approach**
  Roman Popov *et al.*

- **FR-111 Pair creation in low-energy collisions of heavy nuclei beyond the monopole approximation**
  Ilia Maltsev *et al.*

- **TU-28 Electron-positron pair production in space-time-dependent colliding laser pulses**
  Ivan Aleksandrov *et al.*

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Thank you!